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DISCUSSION

BADICI ON INCLOSURES AND THE LIAR PARADOX

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Badici [2008] criticizes views of Priest [2002] concerning the Inclosure Schema and the paradoxes of self-reference. This article explains why his criticisms are to be rejected.

1. Introduction

In *Beyond the Limits of Thought* [Priest 2002] I formulated the Inclosure Schema and argued that it gives the form all the paradoxes of self-reference, that is, that they are all inclosure paradoxes. I argued, on the basis of this, that a uniform solution to all these paradoxes is required. Standard solutions are not of this kind, but a dialetheic solution is. Hence we have an argument for dialetheism. In a recent article Badici contests my views.[1] His objections are essentially two:

1. My argument that all the paradoxes are inclosure paradoxes is question-begging.
2. The Inclosure Schema does not, in any case, give the form of the Liar paradox.

He also argues that:

3. One may think that the Liar paradox is an inclosure paradox because one confuses it with a similar paradox, the 'logical Liar'.

I shall explain, *seriatim*, why I reject all these claims.

2. The Inclosure Schema

The Inclosure Schema is given by the following conditions, which I will refer to as *IS*:

1. Ω exists and $\psi(\Omega)$ (Existence)
2. If $x \subseteq \Omega$ and $\psi(x)$ then:

[1]1. Badici [2008]. Page references are to this unless otherwise stated.

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(a) $\delta(x) \notin x$ (Transcendence)
(b) $\delta(x) \in \Omega$ (Closure)

An inclosure paradox arises when for some δ (the diagonalizer), Ω, and ψ, there are principles which appear to be true (or *a priori* true) and which entail the *IS* conditions.[2] In some paradoxes, the condition $\psi(x)$ is the vacuous condition $x = x$.

Following Ramsey, Badici divides up the paradoxes of self-reference into logico-mathematical paradoxes (group A), and the paradoxes of thought–language–symbolism (group B). As a warm-up objection, Badici says [586]:

> the function [*sic*] ψ has been added to the schema in order to ensure that the diagonalizer is defined for paradoxes in group B. For semantic paradoxes, $\delta(x)$ cannot be straightforwardly defined for all subsets of Ω. ...
>
> It should be noticed that the need to add a restriction ... raises a worry with respect to Priest's uniform account ... [It] appears to be a mere *ad hoc* addition that does not play any role in Russell's paradox, while ... [it] plays a genuine role in the semantic paradoxes.

This is factually incorrect. What I called Kant's Antinomy [Priest 2002: 131] uses the notion of thought, and so is in group B, according to Ramsey; in this, ψ is the vacuous condition. In the liar paradox, as phrased using propositions, rather than sentences, the Heterological paradox, phrased using properties rather than predicates, and Grim's paradox about the proposition that all propositions are not about themselves, ψ is the vacuous condition [Priest 2002: 230]; yet these are all group B. Moreover, 'Gödel's paradox' concerning the sentence which 'says of itself that it is not provable' does require ψ, even though, since it contains only 'logical and mathematical' terms, it is group A.

3. Begging the Question

Now to Badici's first main charge. He notes that if one requires the assumption of dialetheism to establish that all the paradoxes fit the Inclosure Schema, then to use this fact as part of an argument for dialetheism begs the question. This is entirely correct. We must distinguish, as Badici does, between an explanation of why the paradoxes arise and a strategy for how to handle them.[3] The Schema is intended to be the former; it presupposes no particular solution [Priest 2002: 279].

What it does is to characterise the form of a certain family of arguments for paradox. I say 'paradox', not 'dialetheia'.[4] It is not required that the

[2] The *prima facie* nature of the *IS* conditions was not spelled out as clearly as I would now wish in the first, but is quite explicit in the second, edition of *Beyond the Limits of Thought* [2002: 277].

[3] I was not very clear about the distinction [Priest 2002]. Thanks to Badici for seeing its importance.

[4] In [2002: 9.5 and 17.6], I often used the ambiguous term 'contradiction', where it would have been better to use 'paradox'.

arguments entailing the conditions be sound. Indeed, the Inclosure Schema is a modification of a schema proposed by Russell which, according to him, too, captured the form of the paradoxes of self-reference. (See Priest [2002: 9.2].) Russell did not endorse the soundness of the *IS* conditions: he was certainly no dialetheist.

So why does Badici say that applying the schema in the way I do begs the question? For a start, he says [588] that *I* think that an explanation of the paradox presupposes a solution. Apart from the fact that what I think about the matter is irrelevant, this is just not my view. Having an explanation of why the paradoxes arise no more presupposes a solution to them than having an explanation of why global warming arises presupposes a solution to that.[5]

More importantly, he argues [588f]:

> Using [the claim that the Inclosure Schema is the structure that underlies both kinds of paradox] to reject a non-uniform solution to a certain paradox (i.e., a solution that cannot be extended to the paradoxes in the other category) would amount to begging the question against the proponent of that solution, because one who entertains a non-uniform solution to a certain kind of paradoxes would not acknowledge that logical and semantic paradoxes are both generated by the same underlying structure.

Now, there is, of course, a sense in which the argument begs the question, though it is quite trivial. If you hold some view, and I give an argument against it, *any* premise of the argument which you have not already endorsed can be taken to beg the question in this sense. The important question is whether there are plausible independent grounds for the premise in question—in this case, that the Schema delivers the form of all the paradoxes.

Moreover, once one sees how all the paradoxes fit neatly into the same framework, the thought that their form has been captured is, it seems to me, difficult to resist. We see *why* the operation of the diagonalizer is bound to produce a contradiction when the limit is reached. What Badici needs, then, is some independent arguments as to why the Schema does not give the form of all the paradoxes. This is essentially what Badici attempts with his next charge. We will turn to this in a moment. Before I do this, let me make a few further comments.

Questions of form are notoriously hard. Showing that two arguments have the same essential form is always a sensitive matter. (Think of: 'John is not bald, so someone is not bald' and 'The King of France is not bald, so someone is not bald'.) To show that a pattern captures the form of some phenomenon, we need to show that the pattern captures its essence, that the pattern is not an 'accidental' one. What this means in the present case, is that it is the pattern which explains *why* the paradox arises. A way in which this might *not* be the case is as follows: suppose that we have any old paradox of the form $\Omega \in \Omega$ $(\wedge\Omega \notin \Omega$. We could define a diagonalizer simply as the function, f, such that for $x \subseteq \Omega, f(x) = \Omega$. This would give Transcendence and Closure, but it would

[5]He gives two quotations from me suggesting that I think this. I forgo the pedantic task of going through them and pointing out why they do not do so.

clearly be artificial. And to claim that this paradox arises because it fits the Schema would indeed beg the question. It is the other way around: the Schema is satisfied because we have a paradox. For the Schema to be explanatory of inclosure paradoxes, then, we need that the diagonalizer in the Schema be 'genuine', that it truly give the 'mechanism' operating here—though articulating exactly what this means is no easy project.[6]

Badici takes me to task over this matter [589]:

> [Priest's] explanation of why the Inclosure Schema is the schema that underlies the logical and semantic paradoxes (which is needed to guarantee that the solutions are of the same kind) is misleading because it comes from reading too much into the comparison between a diagonalizer and a mechanism that generates an effect.

This, I am afraid, is a misunderstanding. The comparison between the diagonalizer and a mechanism is not an explanation of why the Schema underlies the paradoxes. It explicates what it is for the *IS* conditions to capture the essence of a phenomenon, rather than an accident. He continues [loc. cit.]:

> According to [Priest] 'all inclosure contradictions [i.e., paradoxes] are generated by the same underlying mechanism'; also, the diagonalizer 'manages to "lever itself out" of a totality' [Priest 2002: 289] or to 'operate on a totality of objects of a certain kind to produce a novel object of the same kind [ibid. 136]. Nevertheless, the comparison becomes illegitimate if one puts too much weight into the words 'generate' and 'produce': the diagonalizer would be conferred with the magic power to make inconsistent principles that are otherwise innocuous and to bring novel objects into existence. Of course, none of these powers can be attributed to the diagonalizer.

With all respect, this is another confusion: it takes an analogy for a literal truth. Of course, the diagonalizer does not *literally* lever itself out. This is a metaphor; and no worse for that. One might equally speak of the function which maps any set of ordinals to its least upper bound as producing a cap for the set. None of this attributes the function any 'magic power'—of bringing objects into existence, or of any other kind.

4. The Form of Paradox

I turn to Badici's second charge. This is to the effect that the Inclosure Schema does not give the form of the Liar paradox. He offers three reasons for this. The first is as follows [590]:

> Part of the structure that generates the Liar paradox is, according to Priest, the assumption that there is a set of true sentences of English [Ω]. It is undeniable

[6] All this is discussed at greater length in Priest [2002: 9.5].

> that the existence of a set of true sentences of English has no less *prima facie* intuitive support than the thesis that there is a set of all sets that are not members of themselves. Nevertheless, there is an important difference: the Liar paradox survives after [this assumption] is dropped, while the logical paradoxes do not.

Now, first, all the Schema requires is that the *IS* conditions be *prima facie* true, which the existence of the set in question is, as Badici says in this quote—and once more as well [595]: '... it is intuitively true that there is a set of true sentences of English, which is represented by the predicate "true sentence of English".'

Next, though it is true that one can formulate the paradox without this assumption, it is equally true that one can formulate it simply in terms of the set of true sentences, without the assumption that this set can be defined, and so without the assumption that there is a predicate 'is true'. So the paradox 'survives' after dropping this assumption too. There are many ways of formulating the liar paradox in English. The point is that whether one uses a set or a predicate, essentially the same thing is going on intuitively.

A related objection was made earlier by Tennant and Kroon, and I replied to it briefly in Priest [2002: 279], pointing out the intuitive connection between being true and being a member of the set of true sentences. Badici finds the reply unconvincing for two reasons.[7] The first is that the connection 'should not be taken as granted' [591]. I am not exactly sure what 'taken as granted' is supposed to mean here. But it is intuitively true,[8] as Badici himself says; and this would seem to be quite enough. I also argue that it is reasonable to assume it in the context [2002: 279ff]. This is hardly taking it for granted. The second reason is that 'it is not clear why the thesis that there is a totality of true sentences of English should be made part of the schema' [591]. Again, I am not entirely sure what this means, but I take it to be a reiteration of the point that the Liar can be formulated without referring to the set of true sentences, which I have already addressed.

Badici's second reason for the Inclosure Schema's not giving the form of the Liar paradox is that the diagonalizer, in the case of this paradox, does not really depend on its argument (input).[9] This would make it like the function f in the last section. In the Liar, δ is defined as a function which applies to any definable set of sentences, x, to give a sentence of the form

[7]He also glosses it [591] as 'if one is willing to talk about the true sentences of English, one is thereby committed to there being a set of true sentences of English'. This is clearly not a fair paraphrase.

[8]As I put it: 'No one would ever have doubted this connection, had it not been for the fact that it gives rise to contradiction in certain contexts' [2002: 280].

[9]What Badici actually says [592] is: 'Priest tries to defend the idea that it is the Inclosure Schema that generates (and thus explains) the paradoxes by arguing that the diagonalizers are such that "there is a genuine functional dependence of the value of the function on its argument: the argument is actually used in computing the value of the function" Priest [2002: 136, fn. 18]'. This is not an accurate reading of the footnote, which addresses the question of how to distinguish between a real ψ and a gerrymandered one; and what it says is: 'One way to get some handle on the issue might be to note that in the case of the *bona fide* diagonalisers that we have met, there is a genuine functional dependence of the value of the function on its argument: the argument is actually used in computing the value of the function. This is clearly not the case with the pathological example [f] we noted'.

$<\alpha \notin x>$, where α names that sentence, and angle brackets are an appropriate quotation device. But, says Badici, it is not the set, but its name that occurs in α, since 'x' occurs within quotation marks. Hence, the definition of δ does not deliver a genuine dependence on the set.

Now, for δ to be well defined, x must have a name (which it does in the context, since x is definable). Which name is deployed in α? It doesn't really matter; any name will do.[10] We can enumerate the names and take the first, or we can simply use the Axiom of Choice to select one. Thus, if $S(x)$ is the set of names of x, and θ is a choice function on sets of names, then $\delta(x)$ is a sentence, defined by some self-referential construction, of the form 'α' # '$\notin$' # $\theta(S(x))$. Here, # indicates concatenation. Clearly, there is functional dependence.

In a footnote, Badici considers essentially this point, and avers [593n]:

> ... to prove that this is the desired kind of functional dependency, one would need a principled reason why it is the arbitrarily selected mechanism (rather than any alternative functionally dependent mechanism) that provides the explanation of the paradox.

The point has now changed. It is now, presumably, that though we do have functional dependence, if it can be obtained in different ways, none can be of the essence of the paradox. But the fact that the dependence can be obtained in other ways is irrelevant, in just the same way that which name one uses for the set is irrelevant. Similarly, the self-reference in the Liar paradox can be obtained in many different ways—with demonstratives, descriptions, Gödel coding. It doesn't matter: what is important is that there is self-reference. For that matter, the construction of 'the' Gödel sentence of an axiomatic system of arithmetic is relative to a suitable coding, of which there are many; which one is irrelevant.

Let us turn to Badici's third reason. This is that, though 'it is hard to deny that Transcendence and Closure hold for the set of true sentences, Ω, ... this does not follow *only* from the fact that Ω is definable' [593]. More is required, in the shape of the *T*-schema and bivalence. I agree entirely that these things are required to show that the *IS* conditions hold. Naturally, something must be used to show that the conditions hold! Various principles are required for this: the *T*-schema & Co. are just some of them. In some set theoretic and semantic paradoxes (but not others), various principles about descriptions are required. In the set theoretic paradoxes, principles of set theory are required. The fact that principles not mentioned in the Schema are required cannot, therefore, be held against it.

Nor does the fact that different principles can generate an inclosure show that the paradoxes involved have different essential structures—any more than the fact that an illness can arise under different conditions shows that there are two illnesses. Or, closer to home, as I have already observed, there are many different ways of achieving self-reference, but these do not generate different Liar paradoxes. As I put it, 'focussing on how the

[10] I was not very explicit about the matter [2002:10.2]. This is the reason.

inclosure is achieved is like focussing on how the self-reference is achieved' [2002: 291]; it is irrelevant.

Of course, someone who does not accept the contradiction involved in a paradox of self-reference will have to challenge at least one of the principles involved in generating the *IS* conditions. The principles are therefore going to be disputable; they must be taken as *prima facie*, pending further argument. But that is another matter.

5. Semantic and Logical Liars

I turn, finally, to Badici's third substantial claim. This returns us to the issue of formulating the Liar paradox with a set. Badici distinguishes between the (*semantic*) Liar, a sentence, L, of the form 'L is not true', and the *logical* liar, a sentence L' of the form $L' \notin \{x : x \text{ is true}\}$'. He says [595]:

> the structure of the Liar paradox, as well as the structure of other semantic paradoxes, is different from the structure of logical paradoxes. Consequently, it is perfectly reasonable to expect that the two kinds of paradoxes have different kinds of solution.

Now, this is odd. If the logical Liar is not a semantic paradox, then presumably it is in Ramsey's group A, even though, *contra* Ramsey, it contains an explicitly semantic term. Moreover, if it is in group A, it ought to have the same solution as the others in group A. But in orthodox thinking, it doesn't: the other paradoxes of group A fail because the sets involved are 'too large': the set of all sets, of all ordinals, of all sets that do not contain themselves, are all proper-class size. The set of sentences of English (or of a particular regimented form of it sufficient for the Liar) is countable.

So why does Badici draw the distinction? He says [595]:

> One way to see that the two liars should be sharply distinguished from one another is to notice that the intuitions that lie behind the Logical Liar are parasitic upon those that lie behind the Liar paradox. Absent the intuitions that lead to the Liar paradox, the Logical Liar could be solved by denying that there is a set of true sentences of English (the way one can block other logical paradoxes), or by denying that the set is representable in English.

I find it hard to follow this. The intuition that leads to the Liar paradox is, I take it, primarily the T-schema. And if we were to take this away, then it would precisely be the case that we would not need to deny that there was a set of true sentences, or that this is representable in English. And why on earth does the fact that the intuitions of one paradox are parasitic on those of another show that they are different paradoxes? If it is the same intuition that is driving both, then they are, presumably, the same paradox. Consider the set of ordinals $\{2\alpha : \alpha \text{ is an ordinal}\}$. It does not take much to show that this set is paradoxical—for exactly the same reason that the set of all ordinals is. This is not a reason for taking it to be a paradox different from the Burali–Forti; rather, it is a reason for taking it to be a trivial variation.

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